ET 438a Automatic Control Systems Technology

# LESSON 13: EFFECTS OF NEGATIVE FEEDBACK ON SYSTEM DISTURBANCES





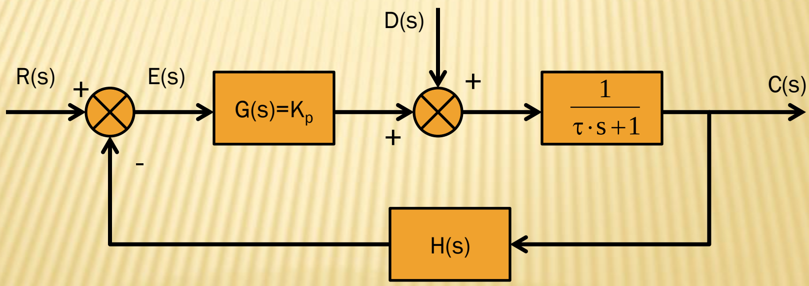
## LEARNING OBJECTIVES

After this presentation you will be able to:

- Reduce a block diagram with multiple inputs using superposition theory.
- Explain how negative feedback reduces the effects of system disturbances with a proportional controller.
- Interpret step-response time plots.
- Use Matlab control toolbox functions to plot the step response of control systems,

# EXAMPLE 13-1 DISTURBANCE REJECTION

**Example 13-1:** Determine the impact of the disturbance function (D(s)) on the control output of the proportional only control shown below. The controller regulates a first-order process with a time constant of $\tau$.



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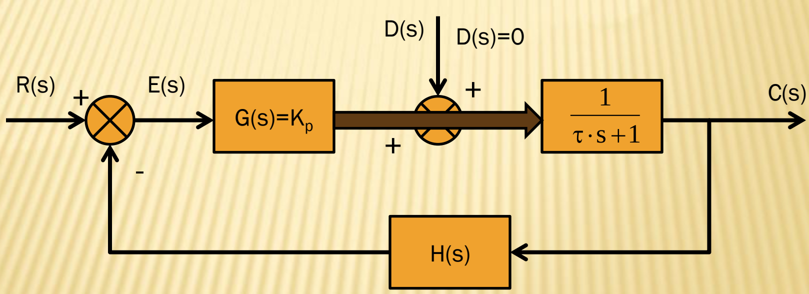
# EXAMPLE 13-1 SOLUTION (1)

Must use superposition for this analysis. Assume D(s), the disturbance on the system is 0 and find output due to R(s) only.

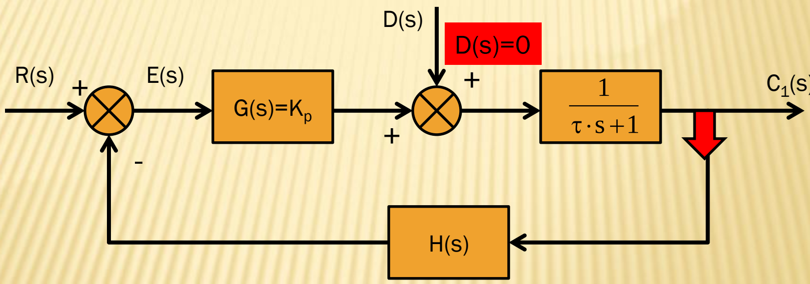



Forward path gain is product of all blocks in upper path

$$G_1(s) = K_p \cdot \frac{1}{\tau \cdot s+1} = \frac{K_p}{\tau \cdot s+1}$$

# EXAMPLE 13-1 SOLUTION (2)

Feedback path is give by H(s). Output $C_1(s)$ is from R(s) input only.
Use feedback reduction formula




$$\frac{C_1(s)}{R(s)} = \frac{G_1(s)}{1+G_1(s)\cdot H(s)} = \frac{\left[\frac{K_p}{\tau\cdot s+1}\right]}{1+\left[\frac{K_p}{\tau\cdot s+1}\right]\cdot H(s)}$$

This is simplified into the following expression





# EXAMPLE 13-1 SOLUTION (3)

Multiply by $(\tau\cdot s+1)$

$$\frac{C_1(s)}{R(s)} = \left[\frac{\tau\cdot s+1}{\tau\cdot s+1}\right]\cdot\left(\frac{\left[\frac{K_p}{\tau\cdot s+1}\right]}{1+\left[\frac{K_p}{\tau\cdot s+1}\right]\cdot H(s)}\right)$$

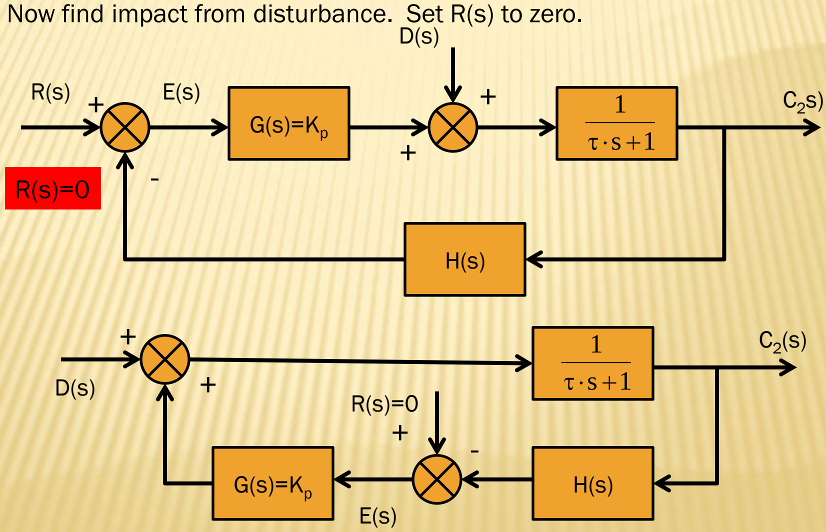
$$\frac{C_1(s)}{R(s)} = \frac{K_p}{(\tau\cdot s+1)+K_p\cdot H(s)} = \frac{K_p}{\tau\cdot s+(1+K_p\cdot H(s))}$$

Call this above function $GH_1(s)$ and save for later use

$$GH_1(s) = \frac{K_p}{\tau\cdot s+(1+K_p\cdot H(s))}$$

# EXAMPLE 13-1 SOLUTION (4)

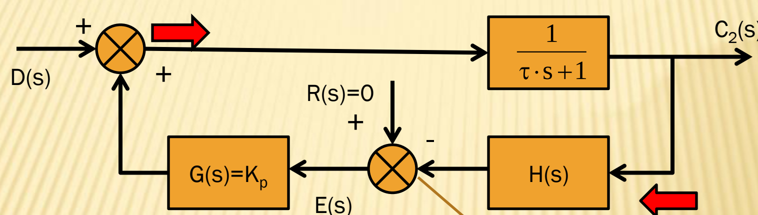
Now find impact from disturbance. Set R(s) to zero.








# EXAMPLE 13-1 SOLUTION (5)




Forward path gain

$$G_2(s) = \frac{1}{\tau \cdot s + 1}$$

Feedback path

$$H_2(s) = K_p \cdot H(s)$$

Negative Feedback

Plug these into the gain formula and simplify

$$\frac{C_2(s)}{D(s)} = \frac{G_2(s)}{1 + G_2(s) \cdot H_2(s)} = \frac{\left[\frac{1}{\tau \cdot s + 1}\right]}{1 + \left[\frac{1}{\tau \cdot s + 1}\right] \cdot K_p \cdot H(s)}$$

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## EXAMPLE 13-1 SOLUTION (6)

Multiple by τs+1

$$\frac{C_2(s)}{D(s)} = \left[\frac{\tau \cdot s+1}{\tau \cdot s+1}\right] \cdot \left( \frac{\left[\frac{1}{\tau \cdot s+1}\right]}{1+\left[\frac{1}{\tau \cdot s+1}\right] \cdot K_p \cdot H(s)} \right)$$

$$\frac{C_2(s)}{D(s)} = \frac{1}{1+\tau \cdot s+K_p \cdot H(s)} = \frac{1}{\tau \cdot s+(1+K_p \cdot H(s))}$$

$$GH_2(s) = \frac{1}{\tau \cdot s+(1+K_p \cdot H(s))}$$

Final output is sum of $C_1$ and $C_2$ so…

$$Y(s) = Y_1(s)+Y_2(s)$$
$$Y(s) = GH_1(s) \cdot R(s)+GH_2(s) \cdot D(s)$$

Plug in the value of $GH_1$ and $GH_2$ to get overall system response

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## EXAMPLE 13-1 SOLUTION (7)

$$Y(s) = \left[\frac{K_p}{\tau \cdot s+(1+K_p \cdot H(s))}\right] \cdot R(s)+\left[\frac{1}{\tau \cdot s+(1+K_p \cdot H(s))}\right] \cdot D(s)$$

I/O Tracking

Disturbance reduced

Notice that the system output is an amplified ($K_p$) version of the input. The disturbance is reduced by the magnitude of the gain $K_p$

The system has 1 pole at $-(1+K_p)/\tau$. If H(s) = 1 and $K_p$ increases Y(s) tracks R(s) and effects of D(s) approach 0. System is faster for higher $K_p$ values.

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# EXAMPLE 13-2: NUMERICAL EXAMPLE

**Example 13-2:** For the system derived in the last example, let $K_p$ = 5, 10, and 20, H(s) = 1, and $\tau$ = 2. Find step response to system to the input and the disturbance. Use Matlab control tool box functions to determine these responses.

Solution: Find transfer function for each of the values of $K_p$

For $K_p$=5

$$Y(s) = \frac{5}{2\cdot s + (1 + 5\cdot 1)}\cdot R(s) + \frac{1}{2\cdot s + (1 + 5\cdot 1)}\cdot D(s)$$

$$Y(s) = \frac{5}{2\cdot s + 6}\cdot R(s) + \frac{1}{2\cdot s + 6}\cdot D(s)$$

One pole in the system at s = -3. (2s+6=0)

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# EXAMPLE 13-2: SOLUTION (2)

Now derive the transfer functions for a $K_p$=10.

$$Y(s) = \frac{10}{2\cdot s + (1 + 10\cdot 1)}\cdot R(s) + \frac{1}{2\cdot s + (1 + 10\cdot 1)}\cdot D(s)$$

$$Y(s) = \frac{10}{2\cdot s + 11}\cdot R(s) + \frac{1}{2\cdot s + 11}\cdot D(s)$$

One pole in the system at s = -5.5. (2s+11=0)

Finally, with $K_p$ = 20

$$Y(s) = \frac{20}{2\cdot s + (1 + 20\cdot 1)}\cdot R(s) + \frac{1}{2\cdot s + (1 + 20\cdot 1)}\cdot D(s)$$

$$Y(s) = \frac{20}{2\cdot s + 21}\cdot R(s) + \frac{1}{2\cdot s + 21}\cdot D(s)$$

One pole in the system at s = -10.5. (2s+21=0)

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# EXAMPLE 13-2: SOLUTION (3)

Generating response plots with Matlab control toolbox functions

Use the following Matlab functions from the command line

sys=zpk(z,p,k) Turns arrays of coefficients into (LTI)
Linear time invarient system called sys.
**Note: Matlab is case sensitive.**
z = array of system zeros
p = array of system poles
k = array of system gains

step(sys) Plots the step response of the system (unit step input).





# EXAMPLE 13-2: SOLUTION (4)

For $K_p$=5

$$Y(s) = \frac{5}{2\cdot s + 6}\cdot R(s) + \frac{1}{2\cdot s + 6}\cdot D(s)$$

Normalize the outputs be dividing both transfer functions by 2 on top and bottom.

$$Y(s) = \left[\frac{\frac{5}{2}}{\frac{(2\cdot s+6)}{2}}\right]\cdot R(s) + \left[\frac{\frac{1}{2}}{\frac{(2\cdot s+6)}{2}}\right]\cdot D(s)$$

$$Y(s) = \left(\frac{2.5}{s + 3}\right)\cdot R(s) + \left(\frac{0.5}{s + 3}\right)\cdot D(s)$$ One pole at -3

This is a two input single output system. The code to generate the step response plots follows.

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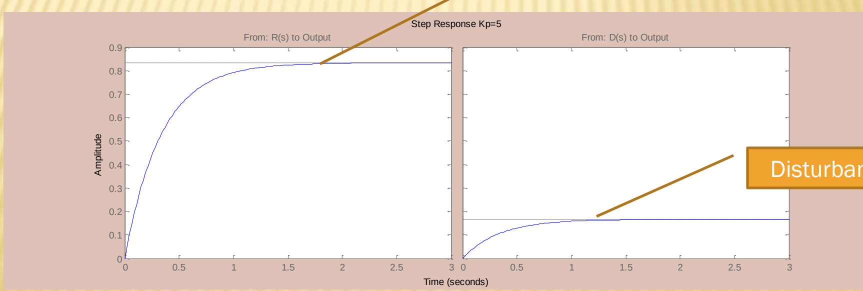
# EXAMPLE 13-2: SOLUTION (5)

**Matlab code $K_p$=5**

```
k = [2.5 0.5]
p = {[-3], [-3]}
z = {[ ], [ ]}
sys = zpk(z,p,k)
step(sys)
```

Desired Output, Y(s)




Disturbance

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# EXAMPLE 13-2: SOLUTION (6)

For $K_p$=10

$$Y(s) = \frac{10}{2\cdot s + 11}\cdot R(s) + \frac{1}{2\cdot s + 11}\cdot D(s)$$

$$Y(s) = \left[\left[\frac{\frac{10}{2}}{\frac{(2\cdot s+11)}{2}}\right]\cdot R(s) + \frac{\frac{1}{2}}{\frac{(2\cdot s+11)}{2}}\right]\cdot D(s)$$

$$Y(s) = \frac{5}{s + 5.5}\cdot R(s) + \frac{1}{s + 5.5}\cdot D(s)$$

One pole at -5.5

**Matlab code $K_p$=10**

```
k = [5 1]
p = {[-5.5], [-5.5]}
z = {[ ], [ ]}
sys = zpk(z,p,k)
step(sys)
```

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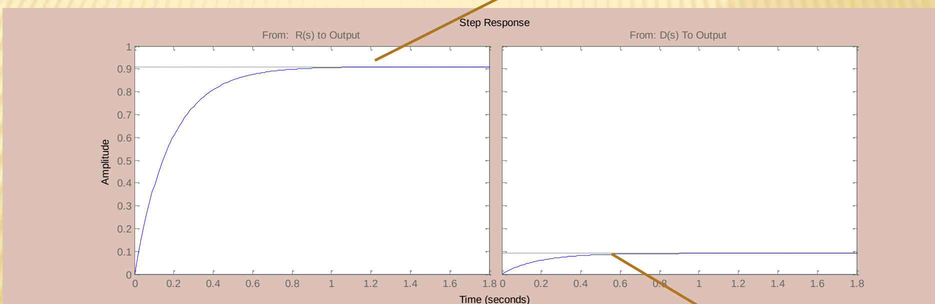
# EXAMPLE 13-2: SOLUTION (7)

Time plots $K_p$=10

Desired Output, Y(s)




Disturbance

Steady-state error of output, Y(s) reduced while disturbance output is reduced

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# EXAMPLE 13-2: SOLUTION (8)

For $K_p$=20

$$Y(s) = \frac{20}{2 \cdot s + (1 + 20 \cdot 1)} \cdot R(s) + \frac{1}{2 \cdot s + (1 + 20 \cdot 1)} \cdot D(s)$$

$$Y(s) = \frac{20}{2 \cdot s + 21} \cdot R(s) + \frac{1}{2 \cdot s + 21} \cdot D(s)$$

$$Y(s) = \frac{\frac{20}{2}}{\frac{(2 \cdot s + 21)}{2}} \cdot R(s) + \frac{\frac{1}{2}}{\frac{(2 \cdot s + 21)}{2}} \cdot D(s)$$

$$Y(s) = \frac{10}{s + 10.5} \cdot R(s) + \frac{0.5}{s + 10.5} \cdot D(s)$$

1 pole at s= -11.5

Increasing value of pole indicates faster response.

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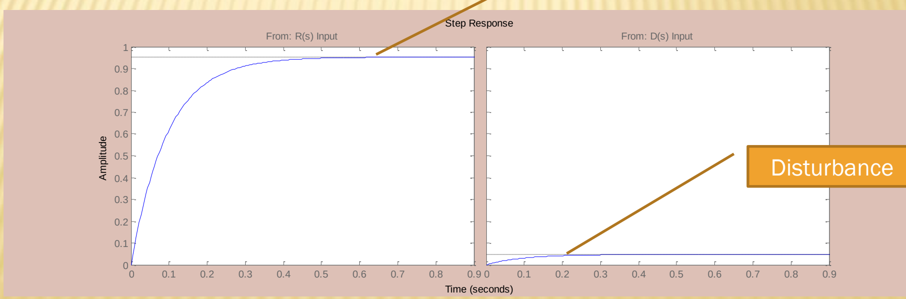
# EXAMPLE 13-2: SOLUTION (9)

Matlab code $K_p$=20

```
k = [10 1]
p = {[-10.5], [-10.5]}
z = {[ ], [ ]}
sys = zpk(z,p,k)
step(sys)
```



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# END LESSON 13: EFFECTS OF NEGATIVE FEEDBACK ON SYSTEM DISTURBANCES